Ole Rømer, the speed of light, the apparent period of Io, the Doppler effect, and the dynamics of Earth and Jupiter

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Ole Rømer's (1676) method of using variations in the apparent period of Jupiter's moon, Io, to demonstrate that the speed of light is finite made use of what we would today call a ''Doppler'' method. He did this 166 years before Christian Doppler described what we now call the *Doppler* effect and the mechanism in 1842. Although the method Rømer conceived is unquestionably valid, his original and only paper on the subject left out much of the detail necessary to determine whether his measurements were adequate to the task of demonstrating the effect he claimed to have observed. Unfortunately, the timekeeping available to Rømer and his colleagues Picard and Cassini, each of whom made some of the observations involved, was, at best, not quite up to the task of measuring the necessary times with sufficient accuracy. Mathematical analysis of the dynamics of the Earth/Jupiter synodic system allows a more thorough analysis of Rømer's work than has previously been made. Rømer's case was built on four ''observations,'' one of which clearly failed, one of which was successful, and two of which were quite questionable.

I. INTRODUCTION

A little over 300 years ago, Ole Rømer, a Danish astronomer working at the Paris Observatory, detected certain variations in the apparent period of revolution of Jupiter's moon, Io, and concluded from his observations and theoretical considerations that the variations were caused by the fact that light has a finite velocity. Rømer's analysis was published in 1676[1] in a paper that has been republished and published in translation many times (e.g., in the Philosophical Transactions of the Royal Society,[2] and in sourcebooks of physics[3] and astronomy[4]). Rømer's work was very important and surprising, and it was greeted with disapproval by his superior at the Observatory (Giovanni Domenico Cassini, who had proposed the same idea a few years earlier and then rejected it[5]), by Robert Hooke, and by many others. Isaac Newton, Edmond Halley, John Flamsteed, and Christian Huygens, on the other hand, more or less immediately accepted Rømer's result.[6] Because of its importance Rømer's initial paper has been extensively discussed in physics and astronomy textbooks, in history of science works, and in journal articles. Some of this discussion has been strongly criticized for its inaccuracy and for its failure to appreciate the context in which Rømer worked.[7,8]

II. RØMER'S HYPOTHESIS

Rømer[4] hypothesized as follows (see my Fig. 1, which is based on Rømer's original):

> ''Now supposing that the Earth when at L, near the second quadrature of Jupiter, has seen the first satellite at the time of its emersion or coming out of the shadow at D; and supposing that about 42.5 hours afterwards, i.e., after a revolution of the satellite, the Earth being at K, the return path at D, it is evident that if light takes time to cross the intervening space LK, the satellite will be seen at D later than it would have been seen if the Earth had remained at L, so that the revolution of the satellite, thus observed by means of its emersions, will be retarded by as much time as the light will have taken to pass from L to K. On the other hand, in the other quadrature FG, the Earth when approaching goes before the light, and the succession of the immersions will appear shortened by as much as those of the emersions had appeared lengthened.''

From this brief, elegant statement, one can see that Rømer had developed a carefully thought out hypothesis regarding what today we would call the Doppler effect on the observed period of Io (the ''first satellite'' of Jupiter) and suggested that observations of Io's period would allow one to determine whether the speed of light is finite.

III. DYNAMICS OF THE SUN–EARTH–JUPITER SYSTEM

Because understanding of the dynamics of the Sun–Earth–Jupiter system is crucial to understanding Rømer's method of determining the speed of light, it is obviously useful to consider a mathematical model of those dynamics. Curiously, these dynamics and the Doppler nature of their effect on the apparent period of Io seem to have been almost entirely overlooked or even misunderstood by many authors (with the exception of Goldstein[9,10] and Debarbat[11]) who have written about Rømer's work. For example, the terms ''synodic year,'' ''opposition,'' ''conjunction,'' ''quadrature,'' and ''Doppler effect'' are almost never even mentioned, let alone discussed, even though they are crucial to understanding Rømer's analysis.

Another curious aspect of the numerous discussions of Rømer's work that have been written over the years is that, as far as I have been able to determine, none of them points out the fact that Rømer actually conceived of and explained what is now universally known as the ''Doppler effect'' 166 years *before* Christian Doppler described and explained the effect in 1842. Perhaps the ''Doppler effect'' ought to be renamed the ''Rømer effect'' on the basis of priority.

Figure 2 shows the relationship of various aspects of the Sun–Earth–Jupiter system. In the diagram, the radii of

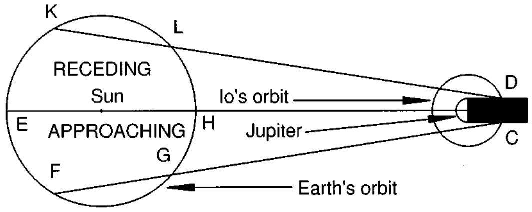

Fig. 1. Modified version of Rømer's original figure. Earth and Io are assumed to be revolving in a counterclockwise direction, with Earth receding from Jupiter at the top of the figure and approaching at the bottom. When Io emerged from Jupiter's shadow at D, Rømer referred to the event as an ''emersion.'' When Io became immersed in Jupiter's shadow at C, he called it an ''immersion.''

Earth's and Jupiter's orbits are approximately in their correct ratio. When Earth is at position E1, Jupiter is in opposition to the Sun; at E5 Jupiter is in conjunction with the Sun. When it is at E4 or E6, Jupiter is in quadrature with the Sun, that is, the angles S–E4–J and S–E6–J are 90 deg. It can also be seen that at quadrature:

$$\angle \mathrm{E4SJ} = \cos^{-1}\left(\frac{\text{Sun–Earth distance}}{\text{Sun–Jupiter distance}}\right) = 78.9°. \qquad (1)$$

In this diagram, if Earth is assumed to be moving counterclockwise; E4 would be the receding quadrature and E6 would be the approaching quadrature.

Figure 2 also shows the basis for calculating the Doppler effect on the apparent period of Io. The distance (D2) from Earth to Jupiter when Earth is at some position E2 can be calculated on the basis of angle A2, which is the angle between the Sun–Earth and Sun–Jupiter lines. When Jupiter is in opposition, that is, when Sun, Earth, and Jupiter lie on the same straight line, that angle is obviously zero. After that, as Earth moves off the Sun–Jupiter line, the angle increases with time, reaching 360 deg in one synodic year. The synodic year for Jupiter is roughly 400 days, but it gets as short as about 395 days and as long as about 403 days. For convenience, in my circular model, I have chosen a synodic year that is exactly equal to 226 of Io's synodic periods (i.e., $226 \times 1.769\,860\,49$ days $= 399.988\,471$ days; the synodic period was taken from the Astronomical Almanac.[12] The elapsed time is then taken in multiples (N) of Io's synodic period, in which case the angle depends on the number of synodic periods of Io that have passed. For each synodic period, angle A2 would increase by about 1.59° (i.e., 360°/226 orbits = 1.59°/orbit).

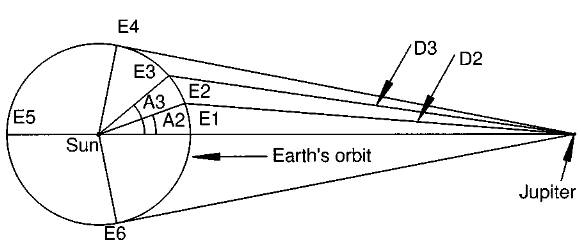

Fig. 2. Geometry of Sun/Earth/Jupiter relations during the course of a synodic year (the time between oppositions). Only Earth's position changes in this diagram of relative motions. If angle A2 is known, the Earth–Jupiter distance (D2) can be calculated with Eq. (2).

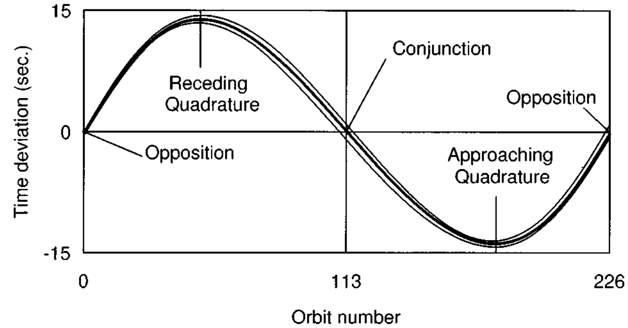

Fig. 3. Expected variation of the deviation (in seconds) of Io's apparent period from its synodic period during a synodic year of 226 orbits of Io. The bold central line is based on a simple, circular, constant-velocity model of Earth's orbital motion. The two flanking lines are based on a more complex, elliptical model of planetary motions. Note that the deviation is near zero for the first orbit, which is completed just after opposition, for the 226th orbit which is completed just before the next opposition, and also for the 113th and 114th orbits which occur just before and just after conjunction. The apparent time deviation reaches its maximum at the quadratures where it amounts to about 14 s for one orbit.

The Earth–Jupiter distance (D2) corresponding to any angle A2 can be calculated as follows:

$$\mathrm{D2} = \sqrt{(R_\mathrm{E} \sin \mathrm{A2})^2 + (R_\mathrm{J} - (R_\mathrm{E} \cos \mathrm{A2}))^2}, \qquad (2)$$

where the Sun–Earth distance (R_E) is 1 AU, and the Sun–Jupiter distance (R_J) is 5.202 803 AU.[13] Subsequently, after Io has made an additional orbit, the angle is A3 and the distance will be D3. The apparent time difference (T) between orbits is then

$$T = \frac{(\mathrm{D3} - \mathrm{D2})}{C}, \qquad (3)$$

where C is the speed of light. This is, of course, the inverse of Rømer's problem; he needed to measure T and solve for C.

Because any variations due to the eccentricity of the planetary orbits are relatively small, my simple model assumes that the orbits are circular and that the orbital velocities of the planets are constant. Consequently, the model is one of Earth's motions *with respect to Jupiter* during what is called a *synodic* year, that is, the time between two oppositions. Use of more complex models would not significantly change the results as will be shown.

Using this simple model, it can be seen that it is relatively easy to calculate what we expect Io's observed period during the course of a synodic year to be as based on Io's known synodic period and the Doppler effect. The results of this calculation are shown in Fig. 3 (bold, central line) as *differences* from the known synodic period and exhibit a nice sinusoidal pattern over the course of the synodic year. Figure 3 also shows the results of a more elaborate calculation (flanking, finer lines) made by a computer program written by Dr. Paul Mohazabbi of the UW-Parkside Physics Department. That program took full account of the effects of the elliptical orbits of Earth and Jupiter and their varying orbital speeds and was run through 17 synodic cycles to obtain representative data on distances and times. As Fig. 3 shows, the effects are rather small, and the differences from the results produced by the simple circular model amount to only a couple of seconds per orbit at most.

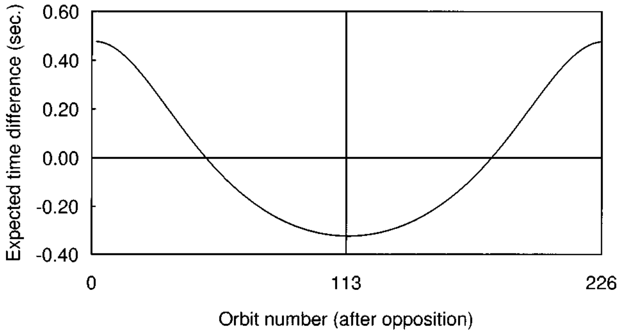

Fig. 4. Variation in the expected time difference between consecutive orbits of Io through a synodic year of 226 orbits (circular model). Note that the differences are all less than about half a second, which explains why early workers such as Rømer chose to work with sets of multiple orbits rather than single orbits.

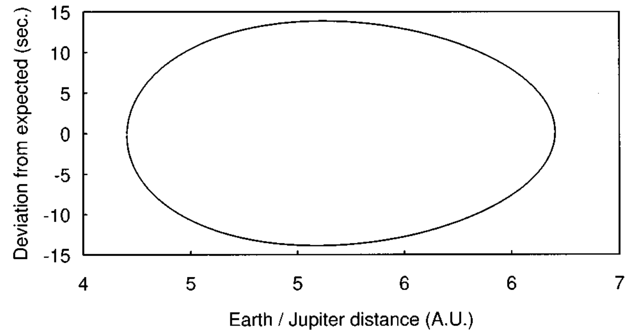

Fig. 6. Variation in the deviation of Io's expected period over the course of a synodic year with the Earth–Jupiter distance (circular model). The deviation is zero when Jupiter is both at its closest and its most distant, and the deviation is greatest at intermediate distances, specifically at the quadratures. These relationships clearly demonstrate that it is *not* distance that controls the deviation.

Figure 3 illustrates a number of points that have been overlooked by most workers. A crucial point is that the observed periods of Io's revolution should gradually increase as Earth moves from opposition to quadrature, then gradually decrease as Earth moves to the other quadrature, and then increase again as Earth moves back to opposition. The difference between observed and ''true'' synodic period would be greatest at quadrature where it would amount to about 14 s for one orbit of Io, that is Io's period of revolution would appear to be about 14 s longer when observed near the receding quadrature than when near opposition and about 14 s shorter when observed at the approaching quadrature; the difference between the two apparent periods would be about 28 s. Also worth noting is the fact that neither ''emersion'' nor ''immersion'' is an instantaneous event; they take about 3.5 min from start to finish, thereby limiting the accuracy with which the period can be measured in a simple, direct manner.

It is also useful to study the time differences between consecutive orbits (Fig. 4), which range from zero up to about half a second. These differences are so small that they are hard to measure and this led Rømer to measure the *total time difference* for a number of orbits rather than the difference for any single orbit. As shown in Fig. 5, my simple mathematical model indicates that the total increase or *lag* time for all orbits on the receding side amounts to about 17 min. Correspondingly, as Earth approaches Jupiter, the total time *gain* is also about 17 min. A crucial point that needs to be understood here, and it is one that has been missed by almost every analyst of Rømer's work, is that the total *time lag* on the receding side equals the *time gain* on the approaching side and that the *difference* between the two is exactly double either value alone.

Since so many authors have made the mistake of claiming that Io's observed period depends on the Earth–Jupiter distance,[8] it is probably also worth taking a look at the relationship of Io's expected period to the Earth–Jupiter distance. Figure 6, which is a graph of the expected deviation from the synodic period versus the distance between Earth and Jupiter, shows clearly that the deviation will be zero when Earth is both closest to and farthest from Jupiter, and that the deviation reaches its maximum at intermediate distances, specifically at the quadratures. A much more useful relationship is that between the expected time deviation for each orbit and the relative velocity between Earth and Jupiter as shown in Fig. 7. This straight-line relationship and the fact that the plot passes through the origin clearly reveal that the relative velocity between Earth and Jupiter is the controlling parameter of Io's changes in apparent period and that the apparent change of Io's period is what we would ordinarily call a *Doppler* effect.

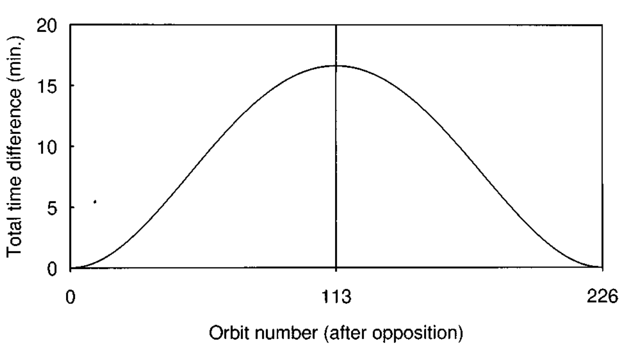

Fig. 5. Variation of the total time difference for Io's expected period during a synodic year of 226 orbits (circular model). Beginning at opposition, the time difference increases to its maximum value of 16.63 min at conjunction and then decreases to zero at the next opposition.

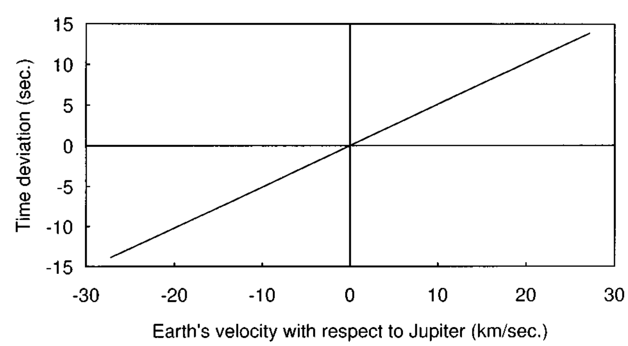

Fig. 7. Variation in expected time deviation for Io's apparent period over the course of a synodic year with Earth's velocity with respect to Jupiter (circular model). The straight-line relationship and the fact that the plot passes through the origin show that the time difference is controlled by the relative velocity of Earth and Jupiter, that is, that this is a *Doppler* effect.

IV. PROBLEMS WITH RØMER'S WORK AND ITS INTERPRETATION

Regrettably, Rømer's celebrated short report on the finite velocity of light was published shortly before it became standard practice to provide careful details about one's observations and methods. In fact, the paper could serve nicely as a case study of the necessity for providing such details. The original paper provides very little information of the kind that one needs to evaluate the work. For example, it does not specify what Rømer used for the synodic period of Io, it does not describe the mathematical calculations he did, nor does it specify the accuracy or precision of the timekeeping involved. It does not even include the dates on which Rømer (or someone else) made the key observations, and it does not attempt to test his hypothesis by comparing his observations in detail to a mathematical model. Instead, the paper provides only the barest *minimum* of information in support of the newly stated hypothesis. Unfortunately, most of Rømer's original papers were lost in the great Copenhagen fire of 1728, but the observations on which he based his claim on the speed of light were subsequently rediscovered in handwritten form by Meyer.[14] These data are discussed later.

Unfortunately, those who have subsequently analyzed Rømer's work have done only slightly better. As far as I have been able to determine, with the exception of Goldstein and his co-workers,[9,10] none of the authors who have written articles on Rømer's work has mathematically analyzed the dynamics of the Earth/Jupiter system to determine the effects of those dynamics on Io's apparent period. Although such an analysis could have been done in Rømer's time by making the same kind of simple assumptions about the nature of planetary motion that I have made in my simple, circular model, it would have been difficult and tedious. However, it is extremely puzzling that this type of analysis has not been done since electronic calculators and microcomputers became so easy to use and so widely available.

Let us first consider Rømer's work as published in Shapley and Howarth.[4] The original Rømer article is only two pages and four paragraphs long, obviously a model of brevity if not of precision. Furthermore, the first two paragraphs are entirely theoretical except for the very final clause in the second paragraph. Among the few quantitative statements made in the first two paragraphs is the following:

> ''Since in the 42 1/2 hours that the satellite takes approximately to make each revolution, the distance between the Earth and Jupiter, in both quadratures, varies at least 210 diameters of the Earth... .''

The figure of 210 Earth diameters is illustrative of the problems Rømer had in attempting to determine the velocity of light because, in fact, *near its quadratures with Jupiter the Earth moves approximately 330 Earth diameters either further from or closer to Jupiter during one of Io's orbits, almost 60% more than Rømer calculated*.

Another significant problem with the Rømer paper is that it does not specify Io's synodic period, which was a crucial value in his analysis. Rømer (and others since then, notably Meyer[14] and Cohen[15]) seems to have attempted to calculate the synodic period by averaging long sets of periods on the receding or approaching sides of Earth's synodic orbit. Aside from the obvious problem that orbital periods measured on the receding side will automatically be longer, and on the approaching side shorter, the average calculated in this manner depends on two other factors, the number of orbits used and which particular sets are used. Figure 8, which is based on the more complicated, elliptical model of Earth/Jupiter dynamics, shows how much variation can be expected from particular sets of N orbits. For example (as illustrated), using a set of 40 orbits can lead to a computed average period that is as little as 3 s too long (or short) or as much as 14 s too long (or short). Despite these difficulties, Meyer[16] shows that Rømer does seem to have come up with a surprisingly accurate value for Io's synodic period, 1d 18h 28m 34 or 36s, as compared to the value given in the Astronomical Almanac[12] of 1d 18h 28m 35.946s.

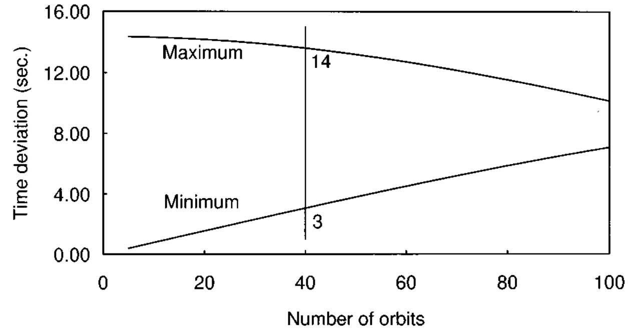

Fig. 8. Variation in deviation of Io's average period from its known synodic period for various sets of N orbits (elliptical model). As shown, for example, the average period for a set of 40 consecutive orbits could be as little as about 3 s too long (or short) or as much as about 14 s too long (or short). This graph demonstrates that determining Io's apparent period by averaging over various sets of N orbits [as Rømer apparently did and later as Cohen (Ref. 15) and Meyer (Ref. 14) did] will not yield consistent results. This graph also shows the range of mathematically permissible values (elliptical model) for the deviation of Io's average period for various sets of N orbits.

V. ACCURACY OF RØMER'S TIMEKEEPING

If one had to rely on the published Rømer paper[1] of 1676 for details on Rømer's investigations, there simply wouldn't be much to work with. Fortunately, however, Meyer[14] discovered a list of more than 50 eclipse times and dates in Rømer's handwriting that provides a basis for more detailed evaluation. However, the list presents us with at least as many questions as answers.

Debarbat[11] carefully analyzed Rømer's handwritten list of eclipses and concluded that: (1) very few of the observations were made by Rømer; (2) some of the eclipse observations were not made at Paris but were made elsewhere and their times corrected for Paris; and (3) a number of transcription errors of both dates and times were made in compiling the list. These facts cast grave doubt on any conclusions drawn on the basis of data from the handwritten list.

Another of the uncertainties involved in using the data from Rømer's handwritten list involves the question of whether the times recorded were ''local sun times'' or ''mean times.'' The difference can amount to as much as about 16 min for one event, and for the elapsed time between two events, the difference can be as great as about 30 min. Both Cohen[15] and Meyer[14] treated Rømer's times as sun times and corrected them with the equation of time. I have followed Cohen and Meyer in this regard. When I first undertook to correct Rømer's times, I used a method described in Seidelmann,[17] which was called to my attention by Evan Gnam of the Astronomy Department at the University of

Wisconsin–Madison. Later,in an effort to get as close as possible to numbers that Rømer probably used, I took my values from Cassini's (1666) table of corrections[18] (a copy of which was graciously provided to me by S. J. Goldstein), which Rømer, himself, must have used, and found little difference between the two methods. All of the corrective values used herein were taken from Cassini's table.

Rømer specified his times in the list to the nearest second, thereby suggesting that he felt his times were accurate to approximately that order of magnitude, and most subsequent workers (including Cohen,[15] Meyer,[14] and Debarbat[11]) have accepted this assessment without comment. Van Helden[19] quotes Picard as saying that the times of the emersions and immersions of Io could be determined to the ''nearest few seconds.'' In 1973 Goldstein and others[9] found that Rømer's times were mostly accurate to within about 2 min, but two years later he and two co-workers concluded that the times had an accuracy of 31.5 s.[10] In the latter case, however, 7 of 57 times were eliminated from consideration *because they had large residual errors*.

The basis for Rømer's timekeeping was the new pendulum clock invented by Huygens in 1656,[20,21] which, it is now thought, had an accuracy of 10–15 s per day.[22] Unfortunately, it is clear that Rømer and his colleagues could not achieve even this latter accuracy routinely.

Another way to evaluate Rømer's timekeeping is to consider the longitudinal difference between Paris and the site of Tycho Brahe's observatory in Uraniborg (then Denmark, now Sweden). It was important to know this difference because Brahe's charts were the best available and they were all keyed to the meridian at Uraniborg.[23] Determining this difference was the reason Picard went to Denmark and ended up bringing Rømer back with him to work at the observatory in Paris. The method used involved Picard's determining the times of the eclipses of the satellites of Jupiter at Uraniborg while Cassini made the same observations in Paris. As it turned out, Rømer made most of the observations at Uraniborg because Picard became ill,[24,6] and later in Paris Rømer determined the longitudinal difference to be 42 min 10 s (of time), a value that he was quite encouraged by according to Meyer.[25] Meyer did not, however, compare Rømer's value to the known modern difference, which is 41 min 28 s; Rømer's value for the time difference was 42 s too high, a clear indication of the limited accuracy of the available timekeeping.

The comparison also illustrates how accurate timekeeping was the key to the method of determining longitude that was being pursued by the astronomers of Rømer's time. The most important reason that Rømer and his French colleagues were so interested in the satellites of Jupiter was that, as suggested originally by Galileo, they hoped to use the immersions and emersions of the satellites as a celestial timekeeping mechanism that would allow the determination of longitude with substantial accuracy. In fact, the astronomical method was used for some time, but eventually mechanical clocks became the basis for the standard method of determining longitude on ships.[26,27,19] The astronomical method was used fairly extensively on land, however. For example, Alexander McKenzie used it in 1793 to determine the longitude of a point on the British Columbia coast after traveling overland across the Peace and Fraser River valleys.[28]

VI. RØMER'S FIRST MEASUREMENT

The Rømer article does describe or at least mention three observations. First, the paper says Rømer measured the period of one orbit of Io when Earth was near its receding quadrature with Jupiter (''L'' in Rømer's original figure and my Fig. 1 here) and the period when Earth was near its approaching quadrature (''G'' in the original and Fig. 1 here). This attempt clearly shows that Rømer had an excellent grasp of how Earth's motion with respect to Jupiter would affect Io's apparent synodic period, because that effect would be at its greatest at the quadratures (see Fig. 3). Unfortunately, Rømer's attempt to measure the difference between the two periods failed when he found that ''no perceptible difference is observed.''[4] *This is a very surprising result*. As Fig. 3 shows, that difference amounts to something like 28 s and should have been measurable if Rømer's timekeeping was anywhere near as good as he and others apparently thought it was. Even more curiously, *this result has attracted absolutely no notice on the part of subsequent workers*, a fact that suggests none of them bothered to calculate what the difference would actually be, nor did they comment on the significance of Rømer's failure to find a difference.

VII. RØMER'S SECOND MEASUREMENT

The second set of observations reported by Rømer, and the set that has received more sustained attention than any other, involved his determination that the time required for 40 orbits when Earth was approaching Jupiter was ''sensibly shorter'' than the time required for 40 orbits when Earth was receding, and that this difference ''amounted to 22 minutes for the entire distance HE, which is double that from here to the Sun''.[4] Curiously, most authors who have since worked on this problem have interpreted the 22-min difference as a time ''lag.'' That is, they have interpreted Rømer to have said that between opposition and conjunction Io's orbits lagged 22 min behind when they were expected to occur. *But this is not what the Rømer paper says*. What the Rømer paper was talking about here was the *difference* between the total apparent times when Earth was receding from Jupiter and the corresponding times when Earth was approaching. The words in the original paper (as translated in Shapley and Howarth[4]) make that clear.

> ''...40 revolutions *observed on the side F* [See my Fig. 1] were sensibly shorter than 40 others *observed on the other side*, and this amounted to 22 minutes for the entire distance HE.'' [emphasis added]

The wording here is crucial. In fact, the time lag when Earth is receding from Jupiter (which we now know to be about 16.63 min) *must equal* the time gain when Earth is approaching, and the *difference* in total time on the receding side from total time on the approaching side must be twice that amount, that is, 33.26 min. So, *Rømer's estimate was about one-third too low*. Most authors who have commented on this value have mistakenly compared Rømer's 22-min figure to the 16.63-min time lag and have concluded that he was about a third too high. This mistake has led many authors to the conclusion that Rømer thought that light takes 11 min to reach Earth from the Sun when, in fact, the words in the original article would lead to a figure of about 5.5 min for that distance and, correspondingly, to a value for the speed of light that is about a third too high. But, it must be emphasized here that Rømer never did try to come up with a value for the speed of light.

Table I. Tabulation of total time deviations and period deviations calculated from the times given in Rømer's handwritten list as published by Meyer (Ref. 14) and Cohen (Ref. 15). The original sun times were changed to mean time values by applying the equation of time correction from Cassini (Ref. 17). Column G shows the total time deviations, and column F shows the range of mathematically ''permissible'' deviations (see Fig. 8). Column H gives the deviations of the periods calculated from observation. Note that there are relatively few instances (boldfaced) where the observed values fall within or close to the mathematically ''permissible'' ranges and there are a number of instances where the observed values deviate substantially from ''permissible'' values. The values in columns G and H are plotted in Figs. 10 and 11. Row 14a gives the figures for the ''due'' date and time (5:25:45) for the 9 November 1676 event that figured so prominently in Rømer's work; row 14b gives the figures for the predicted time of 5:35:45. When compared to mathematically ''permissible'' values the ''due'' time is about 5 min early; the predicted time is, however, quite reasonable, as is the lag between ''due'' time and observed time.

A Set Number	B Begin	C End	D Relative motion	E Number of orbits	F Permissible time deviation (min)	G Total time deviation (min)	H Period deviation (s)
1	10/24/71	1/3/72	A	40	−4 to −9	−15.37	−23
2	10/24/71	1/12/72	A	45	−3 to −10	−17.70	−24
3	10/24/71	2/20/72	A	67	−10 to −14	−20.61	−18
4	11/28/72	3/24/73	A	66	−10 to −14	−742.45	−675
5	11/28/72	2/6/73	A	40	−4 to −9	−735.10	−1103
6	5/12/76	6/13/76	A	18	0 to −4	**−4.77**	**−16**
7	6/9/77	7/9/77	A	17	0 to −4	**−1.75**	**−6**
8	6/9/77	7/25/77	A	26	−1 to −6	**−2.50**	**−6**
9	3/19/71	5/4/71	R	26	2 to 6	−32.56	−75
10	3/7/72	4/29/72	R	30	2 to 7	**−0.73**	**−1**
11	4/18/73	8/4/73	R	61	9 to 13	**10.16**	**10**
12	7/20/75	10/29/75	R	57	8 to 12	**12.78**	**13**
13	8/7/76	11/9/76	R	53	7 to 12	19.21	22
14a	8/23/76	11/9/76	R	44	5 to 10	−1.46	−2
14b	8/23/76	11/9/76	R	44	5 to 10	**8.54**	**12**
15	8/26/77	1/6/78	R	75	7 to 15	**14.30**	**11**
16	8/26/77	11/5/77	R	40	4 to 9	278.48	418

Another problem with the original Rømer paper is that there is no indication in the paper of how Rømer arrived at his 22-min figure. If one studies the data in the handwritten list of observation times and dates,[14,15] one can find two instances where the data indicate that observations were made that in all likelihood spanned 40 orbits of Io while Earth was approaching Jupiter (rows 1 and 5 in Table I here), *but there are no sets of 40 for times when Earth was receding from Jupiter* that were made before Rømer's hypothesis was presented to the Academy of Science in Paris in November of 1676. Furthermore, the observed elapsed times for the two sets of 40 approaching orbits *differ by very nearly 0.5 days or 12 h* as shown below (bottom row). It is worth noting here that Goldstein[10] rejected the time for the 28 November 1672 event as being unreliable and did not include it in his calculations.

Date and time from Rømer's handwritten list	Equation of time effect[18]	Elapsed time (days)
10-24-1671 18h 15m 0s	15.68 m	
1-3-1672 12h 42m 36s	−5.35 m	70.783 77
11-28-1672 5h 37m 5s	11.13 m	
2-6-1673 12h 0m 0s	−14.85 m	70.283 96
Difference		0.499 81

My mathematical models indicate that the greatest possible time deviation for any two sets of 40 orbits on the approaching side is only about 9 min (see Fig. 9), thus casting further doubt on the numbers in the handwritten list of times.

It is possible that Rømer did not mean the number ''40'' specifically, but used ''40'' only to refer to some fairly large number of orbits. The handwritten list of orbital data discovered and published by Meyer[14] and Cohen[15] allows us to check this possibility, but it must be remembered that the numbers in the handwritten list are open to question. Rømer's list consists of the observed times of occurrence of more than 50 ''emersions'' and ''immersions'' of Io. (Some

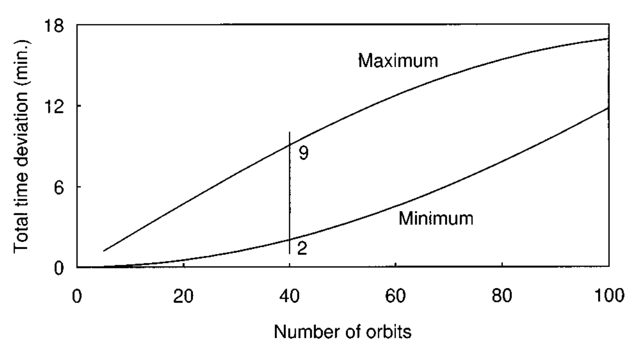

Fig. 9. Variation of total time deviation for various sets of N orbits (elliptical model). For example, sets of 40 orbits may be as much as 9 min too long (or too short) or as few as 2 min too long (or too short), depending on which particular set of N orbits is chosen.

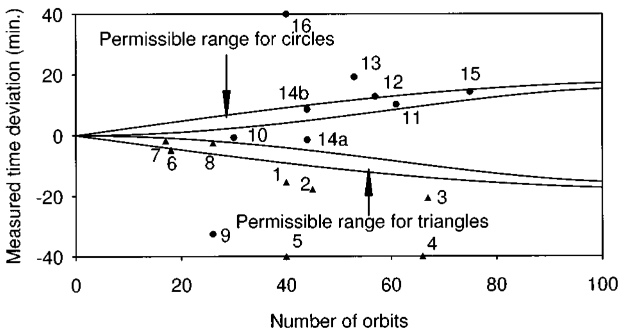

Fig. 10. Graph of total time lags and gains for various numbers of orbits based on Rømer's handwritten list (Refs. 14 and 15) of observed times. The triangles represent values determined when Earth was approaching Jupiter (from Table I); they should all plot below zero and within or at least close to the lower labeled area, which was determined from the mathematical model for elliptical orbits (see Fig. 6). The circles represent values determined when Earth was receding from Jupiter (Table I). *Those values plotted on the top and bottom margins of the graph actually plot well off the scale*. The fact that so few values plot within the ''permissible areas'' is an indication of the problems with Rømer's timekeeping and/or the validity of the data in the handwritten list. It is also clear that Rømer's practice of using large numbers of orbits rather than single orbits to time did not appreciably help the situation.

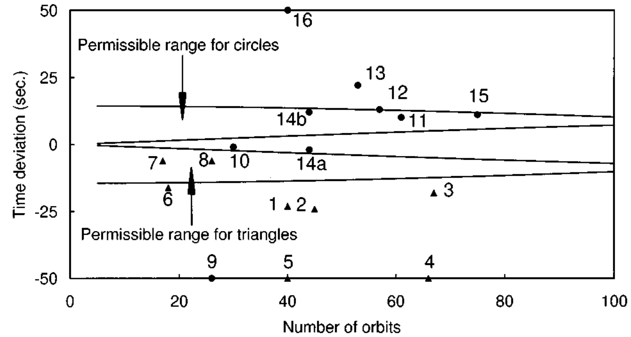

Fig. 11. Graph of Io's period as determined from the numbers provided in Rømer's handwritten list of observed times of emersions and immersions (see Table I here). The circles represent periods determined when Earth was receding from Jupiter (Table I) and should all plot within or close to the upper ''permissible'' range. The triangles represent times determined when Earth was approaching Jupiter and should all plot within or close to the lower permissible range. *The values plotted on the top and bottom margins of the graph actually plot off the scale*. The fact than so few values actually plot within or close to the ''permissible'' areas is a clear indication of problems with Rømer's timekeeping and/or the validity of the data in the handwritten list.

of the numbers are difficult or impossible to read.) The data I used are arranged chronologically in Table I, and they provide a convenient basis for checking the timekeeping available. It is also possible to compare the observations to modern estimates of when Jupiter was in opposition and conjunction during the latter part of the 17th century. The modern estimates were computed from Gingerich and Welther[29] or were furnished directly by Dr. Gingerich from a newly rewritten computer program. All of Rømer's dates of oppositions and conjunctions are in accord with the modern dates.

Rømer's handwritten list does allow one to compute three useful parameters for various pairs of observed events: the number (N) of orbits (column E) completed by Io, the total Doppler time deviation for the set of N orbits (column G), and the deviation of the average observed orbital period from the known synodic period (column H). Column F of Table I gives the approximate range of time deviations that is possible for each particular set of orbits. Careful study of columns G and H clearly shows the problems with these data. Only 8 of the 16 total time deviations (sets 6, 7, 8, 10, 11, 12, 14b, and 15, all boldfaced) fall within or close to the acceptable range of values, and only 8 of the 16 period deviations fall close to or within the allowable range. These results are shown graphically in Figs. 10 and 11.

Both Cohen[15] and Meyer[14] have tinkered extensively with the numbers used by Rømer in an attempt to demonstrate that, if only Rømer had used particular combinations of eclipse times and distances that Earth had moved to or from Jupiter rather than the ones he actually used, he would have come much closer to determining the time it takes light to travel across the radius of Earth's orbit. Frankly, such calculations strike me as rather obvious examples of after-the-fact special pleading.

VIII. RØMER'S THIRD MEASUREMENT

The third ''measurement'' described in the original Rømer paper of 1676[1,4] was the prediction he made at ''the beginning of September'' (of 1676) that an ''emersion'' of Io that was ''due'' on the 9th of November at 5:25:45 would be observed to take place ''ten minutes later than one should have expected in deducing the emersions from those which had been observed during the month of August when the Earth was much nearer Jupiter.'' The Rømer paper says that the emersion was observed on 9 November at 5:35:45 in the evening but, once again in the style of the time, the crucial information needed to evaluate this claim is not provided. Three pieces of information would be needed to evaluate Rømer's ''prediction:'' (1) the precise time and date of the last observed emersion in August, (2) the number of orbits that Io was expected to go through (although this can be calculated), and (3) the precise synodic period of Io that was to be used. *None of these is given in the published work*.

And there is still another problem. Cohen[30] has pointed out that the ''official'' time of the November ''emersion'' as determined by Le Monnier was actually 5:37:49, which would make it an *additional* 2 min later than predicted. Even more puzzling is the fact that in Rømer's own handwritten list of immersions and emersions (reproduced in Cohen[31] and Meyer[14]) the 9 November event is noted as having occurred at 5:45:35, which reverses the minute and second values and makes the observed occurrence $9'50''$ later than the time given in the Rømer paper. My best guess is that this later mistake is an accident of transcription.

Despite these problems, it is possible to evaluate Rømer's prediction. The handwritten list of times discovered by Meyer includes the date (23 August, 1676) and time (8h 11m 13s) of the emersion that would have been the starting point for Rømer's calculation. Also, Owen Gingerich of the Harvard–Smithsonian Center for Astrophysics has graciously used a new computer program developed by him to calculate that the previous opposition took place at approximately 6:48 PM (UT) on 9 July 1676, and the elapsed time between these two events tells us that 25 orbits of Io probably took place during that interval. Then, between 23 August and 9 November, another 44 orbits must have occurred, and my mathematical models suggest that we should expect a total time delay for those orbits (that is, numbers 26 through 59 since opposition) of about 9.5 min, which is almost exactly what Rømer predicted. My Table I shows that Rømer's ''due'' time for the 9 November emersion was about 2 min early (row 14a, column G), but his total time delay for those 44 orbits (10 min) was very close to the 9.5 min that would be expected for orbits 26 through 59. In my estimation, this prediction constitutes Rømer's best claim to having actually demonstrated that the speed of light is finite.

IX. RØMER'S FOURTH MEASUREMENT

Rømer's fourth ''measurement'' was not mentioned in the 1676 paper but was discussed in correspondence with Huygens.[32] In this correspondence Rømer claimed that the average duration between emersions (when Earth is receding from Jupiter) is always greater than the duration between immersions (when Earth is approaching). Unfortunately, because of Rømer's timekeeping problems and because of uncertainty as to the validity of the times and dates in the handwritten list, the case isn't quite so clear.

It is regrettable that Debarbat[11] did not put together an annotated list of Rømer's eclipse dates and times, indicating who made which observations and where they were made, giving the Cassini correction values, and giving the revised mean times. This would have made it possible for later workers to do a better job of evaluating those numbers.

As can be seen by consulting column H of Table I and Fig. 11, although the deviations of the calculated average periods of immersions (rows 1–8) are, in fact, all negative as would be expected, only three of the eight deviations (Nos. 6, 7, and 8) fall within or near the acceptable range, and two of the eight (4 and 5) plot well off the scale in Fig. 11. Furthermore, of the eight emersions (sets 9–16) and one ''due'' emersion, all of which should have positive values in column H, three are negative, and one (No. 9) plots off scale on the negative side. The situation is much better with the remaining six values, only one of which (No. 16) plots too far from the permissible range. Overall, however, only 8 of 16 plotted points in both Figs. 10 and 11 (those that are boldfaced in Table I) fall within or quite close to the range of what is mathematically ''permissible.'' This is definitely a data set whose significance is questionable because of its internal lack of consistency and failure to fit the mathematical models.

X. SUMMARY

It is clear from the analysis provided here that Rømer conceived a theoretically valid method for determining whether the speed of light is finite and even for determining a value for the speed of light. The method he came up with is based on what we call today the ''Doppler'' effect. It could, in fact, be argued that Rømer conceived the Doppler effect 166 years before Christian Doppler did.

Unfortunately for Rømer, the timekeeping available to him was just not quite up to the task of measuring the rather small time differences that is required to demonstrate the effect he was looking for. In support of his claim about the speed of light Rømer specifically cited four ''observations'' that he made or attempted to make to establish his claim. He, himself, acknowledged the failure of his first attempt when he was unable to detect what we now know to be a time difference of about 28 s between orbits of Io at the two quadratures. His second claim, about the 22-m time total difference between receding and approaching orbits, yielded a time that was about a third too low. This, in and of itself however, is not the problem. The real problem is that he did not explain how he came up with the 22-min figure, that his raw data did not include a 40-orbit set when Earth was receding from Jupiter, and that his raw data, taken as a whole (Table I), show so much error that they do not support his claim very strongly at all. Rømer's third ''observation,'' the predicted time for the ''emersion'' of 9 November 1676, turns out to be his strongest piece of supporting evidence, but even it is tainted by timekeeping problems. Rømer's fourth ''observation,'' the claim that averages of immersion periods are always shorter than averages of emersion periods, is of questionable validity because of timekeeping problems and because of uncertainty about the values compiled in the famous ''handwritten list.''

I think it is clear, therefore, that, although Rømer conceived of a valid method for determining that the speed of light is finite and even for determining a numerical value for the speed, he, himself, was not able to measure time accurately enough to show conclusively the validity of his hypothesis, and he never did calculate the speed of light. His method and his data were, however, enough to convince a number of the scientific luminaries of his day that his ideas were valid and correct. Furthermore, we now know that his proposed method was, in fact, valid and that his conclusion that the speed of light is very great but finite was also correct.

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